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Reply by Author to B. Steverding

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THE author is grateful for the opportunity to exchange views on the flow of non-Newtonian fluids with B. Steverding. The paper[1] to which the preceding comments refer contains the solution to a simple problem of two-dimensional, unsteady, isothermal, purely viscous, non-Newtonian boundary-layer flow. The solution was found through a technique for establishing the conditions for similarity (velocity profiles that transform linearly in y). The situation that Steverding is investigating is very interesting and is basically a different problem since the flow is nonisothermal and steady.

The interesting comparison between the two analyses, as the comment points out, is that of the different expressions used to describe the non-Newtonian effect of shear rate. There is, of course, no need to defend the use of the Ostwald-de Waele (power-law) expression, which is phenomenological in nature, as opposed to the Eyring-Bueche expression, which is based on macromolecular considerations. The lack of a theoretical basis for the power-law is conceded initially, and the justification for its use is only in its form, which allows it to be solved explicitly for shear rate. It would seem that the primary consideration in choosing a formulation of the viscous properties for a boundary-layer problem is the accuracy of the formulation in describing the shear rate dependence over the applicable range of flow conditions. In other words, any formulation that describes the measured fluid properties with a prescribed accuracy (and satisfies the mathematical conditions) is appropriate for use in solving a fluid mechanics problem.

As for the temperature dependence of the viscous properties, it is not obvious to the author that the power-law will give wrong results per se for nonisothermal flow, since the coefficient and exponent could be experimentally determined as a function of temperature. The formulations given by Steverding for the temperature-dependence would be particularly interesting, however, if the constants could be predicted theoretically for non-Newtonian fluids, or if the number of empirically determined constants could be reduced.

One question concerning the comment may be worth mentioning for the sake of non-Newtonian fluid nomenclature. The Eyring-Bueche expression is said to provide for rheopectic or thixotropic behavior of viscosity. Thixotropy (or rheopexy) is generally used to describe time-dependent flow effects.[2, 3] Neither the power-law nor the Eyring-Buethe expressions provide for time-dependence. Rather, it appears that (for $k\tau^2 < 1$) the sign of k determines whether the fluid is shear thinning or shear thickening.

It may be of interest to note that the two-dimensional stagnation boundary layer of a power-law non-Newtonian fluid under conditions of steady isothermal flow (considering the convective terms, but not the inertia terms) does give similar solutions.[4] The boundary-layer thickness δ is given by

$$\delta = \eta_\delta \left[\frac{(n+1)(\bar{x})^{n-1}}{2\, R_{0n}\, \alpha^{2-n}}\right]^{1/(n+1)}$$

where the symbols are those defined in Ref. 1. δ is seen to be directly proportional to $(\bar{x})^{(n-1)/(n+1)}$ and inversely proportional to $R_{0n}{}^{1/(n+1)}$. This is compared with δ for stagnation flow of a Newtonian fluid which is not a function of x and is inversely proportional to $R_{0n}{}^{1/2}$.

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Comment on "Conical Shock-Wave Angle"

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SEVERAL approximation formulas for conical shock-wave angles have recently appeared in this Journal.[1, 2] It is the purpose of this note to point out two other equations, both based on the hypersonic similarity rule, which are useful at high values of the hypersonic similarity parameter $M_1 \sin\theta_v$.

Linnell and Bailey[3] first applied the hypersonic similarity law to the Kopal tables[4] and obtained the following formula for the shock-wave half angle θ_w:

$$M_1 \sin\theta_w = 1 - \cos\theta_v + \{1 + [(\gamma + 1)/2]M_1^2 \sin^2\theta_v\}^{1/2} \quad (1)$$

where M_1 is the freestream Mach number, θ_v the cone semi-vertex angle, and γ the ratio of specific heats. For $M_1 \geq 6$ and $\theta_v < 50°$, Eq. (1) agrees to within 5% with the data of Ref. 4. Notice that, for large values of M_1 and small values of $\cos\theta_v$, Eq. (1) approaches Eq. (2) of Ref. 1. However, under these conditions, say for $M_1 \gtrsim 10$ and $\theta_v \gtrsim 40°$, the air behind the shock-wave will become dissociated and these equations will no longer describe the shock geometry correctly. Instead, a second approximation equation derived from the numerical solutions of Ref. 5 can be used.

When the conical flow solutions of Ref. 5 for dissociated air were obtained, it was pleasantly surprising to find that the results correlated extremely well with the hypersonic similarity parameter $M_1 \sin\theta_v$, even more so than for the case of a perfect gas. As a result, the following estimation formula has been useful in predicting values for the shock-wave angle for cases when the air behind the shock is expected to be in dissociation equilibrium and where tabulated values are not available:

$$M_1 \sin\theta_w = (0.39 - 0.01 \log p_1) + (1.03 + 0.005 \log p_1) M_1 \sin\theta_v \quad (2)$$

where p_1 is the ambient pressure in atmospheres.

Equation (2) agrees with the exact numerical calculations of Ref. 5 to within ±1%, for $M_1 \sin\theta_v \geq 6$.

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